

Mathematics with Applications

in the Management, Natural, and Social Sciences

ELEVENTH EDITION



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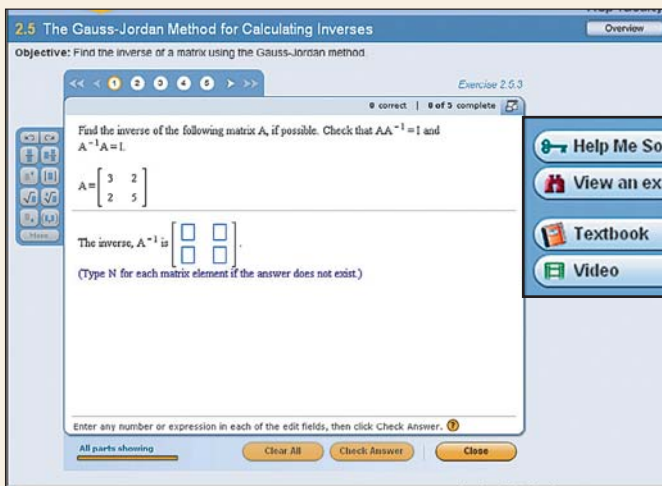
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Mathematics with Applications

IN THE MANAGEMENT, NATURAL, AND SOCIAL SCIENCES

ELEVENTH EDITION

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*To G.W. and R.H., without your support
this work would not be possible.*

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Preface

Mathematics with Applications is an applications-centered text for students in business, management, and natural and social sciences. It covers the basics of college algebra, followed by topics in finite mathematics and concluding with a treatment of applied calculus. The text can be used for a variety of different courses, and the only prerequisite is a basic course in algebra. Chapter 1 provides a thorough review of basic algebra for those students who need it. The newly added Prerequisite Skills Test (for Chapters 1–4) and Calculus Readiness Test (for Chapters 11–14) at the front of the text can help determine where remediation is needed for students with gaps in their basic algebra skills.

It has been our primary goal to present sound mathematics in an understandable manner, proceeding from the familiar to new material and from concrete examples to general rules and formulas. There is an ongoing focus on real-world problem solving, and almost every section includes relevant, contemporary applications.




New to This Edition



With each revision comes an opportunity to improve the presentation of the content in a variety of ways. This revision is no exception as we have revised and added content, updated and added new applications, fine-tuned pedagogical devices, and evaluated and enhanced the exercise sets. In addition, both the functionality of MyMathLab and the resources within it have been greatly improved and expanded for this revision. These improvements were incorporated after careful consideration and much feedback from those who teach this course regularly. Following is a list of some of the more substantive revisions made to this edition.

- **Prerequisite Diagnostic Tests:** As a way to address the often weak algebra skills that students bring to this course, two diagnostic exams are provided just prior to Chapter 1. These two diagnostics can be used in a variety of ways, depending on the expectations and goals of the course. The Prerequisite Skills Test assesses student understanding of the core skills students should have prior to starting Chapter 1. The Calculus Readiness Test is designed to diagnose any missing prerequisite algebra skills prior to starting Chapter 11. Solutions are provided in the back of the text so students can self-remediate as needed, saving valuable class time. *Getting Ready* quizzes in MyMathLab are also available as automated versions of these diagnostics.
- **Additional Figures:** An additional six graphical figures were added to Section 2.2 to help illustrate slope as part of writing the equation of a line.
- **Quadratic Functions:** In the previous edition, there were two separate sections (3.4 and 3.5) for quadratic functions and applications of quadratic functions. These two sections have been consolidated into one section for ease of teaching.
- **Implicit Differentiation and Related Rates:** By popular demand, two sections have been added: implicit differentiation and related rates. These sections include step-by-step instructions for students to find the derivative of an implicitly defined function and to find a rate of change given that two quantities are related to each other and also changing over time. Examples and exercises include applications to business such as the rates of change of the number of employees and revenue of a corporation.
- **Case Studies:** The popular culminating activities at the end of every chapter have been expanded to provide options for **extended projects** for individual students or more extensive **group work**. In addition, three of the case studies are completely new and five have been updated significantly.

- A careful and thorough review of the **correlation between examples and exercises** has been completed to ensure that students have examples to refer to when working homework exercises. A table at the start of the answers for the Review Exercises (in the Answers to Exercises section in the back of the book) tells students in which section each of the review exercises is covered.
- The **MyMathLab** course that accompanies this text has been markedly improved and expanded to include even greater coverage of the topics in this course through more exercises, videos, animations, and PowerPoint slides. In addition, answer tolerance for exercises has been checked and adjusted when necessary, and prerequisite diagnostic tools are available for personalized assessment and remediation. See the Supplements portion of this preface for more details on MyMathLab.
- **Annotated Instructor's Edition:** An invaluable resource for instructors, the Annotated Instructor's Edition (AIE) contains answers to the exercises right on the exercise set page (whenever possible). In addition, *Teaching Tips* are included in the margins to provide helpful hints for less experienced instructors.
- **Graphing Calculator** screen shots have been updated to represent the newest TI calculator, the TI-84+ C. These visuals often prove valuable even to students not using graphing calculators.
- The **exercises** are now better paired so that the even exercises require similar knowledge as the corresponding odd exercise. In addition, approximately 21 percent of the 5,436 exercises in the text are new or updated.
- Approximately 22 percent of the 641 **examples** in the text are new or updated. Many examples and exercises are based on real-world data.
- **Student Learning Objectives** for every section of the text are provided at the back of the text. These objectives describe what students should understand and be able to do after studying the section and completing the exercises.
- An **Index of Companies, Products, and Agencies** is provided to increase student interest and motivation.

Continued Pedagogical Support

- **Balanced Approach:** Multiple representations of a topic (symbolic, numerical, graphical, verbal) are given when appropriate. However, we do not believe that all representations are useful for all topics, so effective alternatives are discussed only when they are likely to increase student understanding.
- **Strong Algebra Foundation:** The text begins with four thorough chapters of college algebra that can be used in a variety of ways based on the needs of the students and the goals of the course. Take advantage of the content in these chapters as needed so students will be more successful with later topics and future courses.
- **Real-Data Examples and Explanations:** Real-data exercises have long been a popular and integral aspect of this book. A significant number of new real-data examples and exercises have also been introduced into the text. Applications are noted with the type of industry in which they fall so instructors or students can focus on applications that are in line with students' majors. (See pages 190–192.)
- **The Checkpoint Exercises** (marked with icons such as ) within the body of the text provide an opportunity for students to stop, check their understanding of the specific concept at hand, and move forward with confidence. Answers to Checkpoint Exercises are located at the end of the section to encourage students to work the problems before looking at the answers. (See pages 184, 192.)
- **Cautions**  highlight common student difficulties or warn against frequently made mistakes. (See page 204.)
- **Exercises:** In addition to skill-based practice, conceptual, and application-based exercises, there are some specially marked exercises:
Writing Exercises  (see page 216)

Connection Exercises  that relate current topics to earlier sections (see pages 253 and 262) Practice Exercises from previous CPA exams (see page 242), and Technology Required Exercises  (see page 263).

- **Example/Exercise Connection:** Selected exercises include a reference back to related example(s) within the section (e.g., “See Examples 6 and 7”) to facilitate what students do naturally when they use a book—i.e., look for specific examples when they get stuck on a problem. Later exercises leave this information out and provide opportunities for mixed skill practice.
- End-of-chapter materials are designed to help students prepare for exams and include a **List of Key Terms and Symbols** and **Summary of Key Concepts**, as well as a thorough set of **Chapter Review Exercises**.
- **Case Studies** appear at the end of each chapter and offer contemporary, real-world and up-to-date applications of some of the mathematics presented in the chapter. Not only do these provide an opportunity for students to see the mathematics they are learning in action, but they also provide at least a partial answer to the question, “What is this stuff good for?” These have been expanded to include options for longer-term projects if the instructor should choose to use them.


Technology

In our text, we choose to support a variety of different technologies in a way that provides greater understanding and encourages exploration of the tools students have at their disposal. The technology that is supported by this text includes graphing calculators, spreadsheets, and a variety of resources within MyMathLab.


MyMathLab: This text is not just supported by MyMathLab (MML), but rather it is enriched and expanded upon within MML. Fortunately, today’s students are well-equipped to take full advantage of resources beyond the text, and the MML course for this text is especially valuable to the typical student.

- Concepts that are difficult to describe in print can be shown dynamically through video or animation, providing a visual that students may otherwise miss.
- Topics that are particularly challenging to students can be practiced, assessed, reviewed, and practiced some more through algorithmically generated homework or videos.
- Prerequisite skills that students lack can be assessed and then addressed on an individual basis using quick, pre-built assessments and personalized homework.

Although use of MML with this text is completely optional, students stand a greater chance of success if they take advantage of the many resources available to them within MML. More details as to all that MML can do are provided in the Supplements portion of this preface.

Graphing Calculators and Spreadsheets: It is assumed that all students have a calculator that will handle exponential and logarithmic functions. Beyond that, however, graphing calculator and spreadsheet references are highlighted in the text so that those who use the technology can easily incorporate it and those who do not can easily omit it. Examples and exercises that require some sort of technology are marked with the icon , making it obvious where technology is being included.

Instructors who routinely use technology in their courses will find more than enough material to satisfy their needs. Here are some of the features they may want to incorporate into their courses:

- **Examples and Exercises marked with ** A number of examples show students how various features of graphing calculators and spreadsheets can be applied to the topics in this book. Exercises marked with the same icon give students a chance to practice particular skills using technology.
- **Technology Tips:** These are placed at appropriate points in the text to inform students of various features of their graphing calculator, spreadsheet, or other computer programs. Note that Technology Tips designed for TI-84+ C also apply to the TI-84+, TI-83, and TI-Nspire.

- **Appendix A: Graphing Calculators:** This appendix consists of a brief introduction to the relevant features of the latest TI-84+ C graphing calculator. An outline of the appendix is on page 859, and the full appendix is available online in MyMathLab and at www.mathstatsresources.com.
- A **Graphing Calculator Manual** and **Excel Spreadsheet Manual** are also available within MyMathLab. These manuals provide students with the support they need to make use of the latest versions of graphing calculators and Excel 2013.

Course Flexibility

The content of the text is divided into three parts:

- College Algebra (Chapters 1–4)
- Finite Mathematics (Chapters 5–10)
- Applied Calculus (Chapters 11–14)

This coverage of the material offers flexibility, making the book appropriate for a variety of courses, including:

- Finite Mathematics and Applied Calculus (one year or less). Use the entire book; cover topics from Chapters 1–4 as needed before proceeding to further topics.
- Finite Mathematics (one semester or two quarters). Use as much of Chapters 1–4 as needed, and then go into Chapters 5–10 as time permits and local needs require.
- Applied Calculus (one semester or quarter). Cover the precalculus topics in Chapters 1–4 as necessary, and then use Chapters 11–14.
- College Algebra with Applications (one semester or quarter). Use Chapters 1–8, with Chapters 7 and 8 being optional.

Pearson regularly produces custom versions of this text (and its accompanying MyMathLab course) to address the needs of specific course sequences. Custom versions can be produced for even smaller-enrollment courses due to advances in digital printing. Please contact your local Pearson representative for more details.

Chapter interdependence is as follows:

<i>Chapter</i>	<i>Prerequisite</i>
1 Algebra and Equations	None
2 Graphs, Lines, and Inequalities	Chapter 1
3 Functions and Graphs	Chapters 1 and 2
4 Exponential and Logarithmic Functions	Chapter 3
5 Mathematics of Finance	Chapter 4
6 Systems of Linear Equations and Matrices	Chapters 1 and 2
7 Linear Programming	Chapters 3 and 6
8 Sets and Probability	None
9 Counting, Probability Distributions, and Further Topics in Probability	Chapter 8
10 Introduction to Statistics	Chapter 8
11 Differential Calculus	Chapters 1–4
12 Applications of the Derivative	Chapter 11
13 Integral Calculus	Chapters 11 and 12
14 Multivariate Calculus	Chapters 11–13

Student Supplements

Student's Solutions Manual

- By Salvatore Sciandra, Niagara County Community College
- This manual contains detailed carefully worked out solutions to all odd-numbered section exercises and all Chapter Review and Case exercises.

ISBN 13: 978-0-321-92492-6

ISBN 10: 0-321-92492-4

Graphing Calculator Manual (downloadable)

- By Victoria Baker, Nicholls State University
- Contains detailed instruction for using the TI-83/TI-83+/TI-84+ C.
- Instructions are organized by topic.
- Available in MyMathLab.

Excel Spreadsheet Manual (downloadable)

- By Stela Pudar-Hozo, Indiana University—Northwest
- Contains detailed instructions for using Excel 2013.
- Instructions are organized by topic.
- Available in MyMathLab.

Instructor Supplements

Annotated Instructor's Edition

- The AIE contains answers to all exercises in the text on the exercise set page whenever possible.
- In addition, Teaching Tips are provided in the margin to give less-experienced instructors a cue as to where students typically struggle.

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Instructor's Solutions Manual (downloadable)

- By Salvatore Sciandra, Niagara County Community College
- This manual contains detailed solutions to all text exercises, suggested course outlines, and a chapter interdependence chart.
- Available through www.pearsonhighered.com/irc or in MyMathLab.

Printable Test Bank (downloadable)

- By David Bridge, University of Central Oklahoma
- This test bank includes four alternate tests per chapter that parallel the text's Chapter Tests.
- Available through www.pearsonhighered.com/irc or in MyMathLab.

Media Supplements

MyMathLab® Online Course (access code required)

MyMathLab delivers **proven results** in helping individual students succeed.

- MyMathLab has a consistently positive impact on the quality of learning in higher education math instruction. MyMathLab can be successfully implemented in any environment—lab-based, hybrid, fully online, traditional—and demonstrates the quantifiable difference that integrated usage has on student retention, subsequent success, and overall achievement.
- MyMathLab's comprehensive online gradebook automatically tracks your students' results on tests, quizzes, homework, and in the study plan. You can use the gradebook to quickly intervene if your students have trouble, or to provide positive feedback on a job well done. The data within MyMathLab is easily exported to a variety of spreadsheet programs, such as Microsoft Excel. You can determine which points of data you want to export, and then analyze the results to determine success.

MyMathLab provides **engaging experiences** that personalize, stimulate, and measure learning for each student.

- **Continuously Adaptive:** Adaptive learning functionality analyzes student work and points them toward resources that will maximize their learning.
- **Exercises:** The homework and practice exercises in MyMathLab are correlated to the exercises in the textbook, and they regenerate algorithmically to give students unlimited opportunity for practice and mastery. The software offers immediate, helpful feedback when students enter incorrect answers.
- **Chapter-Level, Just-in-Time Remediation:** Students receive remediation only for those skills that they have not yet mastered through *Getting Ready* diagnostics and content, and personalized homework.

- **Multimedia Learning Aids:** Exercises include guided solutions, sample problems, animations, videos, and eText clips for extra help at point-of-use.
- **Expert Tutoring:** Although many students describe the whole of MyMathLab as “like having your own personal tutor,” students using MyMathLab do have access to live tutoring from Pearson, from qualified math and statistics instructors who provide tutoring sessions for students via MyMathLab.

And, MyMathLab comes from a **trusted partner** with educational expertise and an eye on the future.

- Knowing that you are using a Pearson product means knowing that you are using quality content. That means that our eTexts are accurate and our assessment tools work.
- Whether you are just getting started with MyMathLab, or have a question along the way, we’re here to help you learn about our technologies and how to incorporate them into your course.

To learn more about how MyMathLab combines proven learning applications with powerful assessment, visit www.mymathlab.com or contact your Pearson representative.

MyMathLab[®] Ready to Go Course (access code required)

These new Ready to Go courses provide students with all the same great MyMathLab features, but make it easier for instructors to get started. Each course includes pre-assigned homework and quizzes to make creating a course even simpler. In addition, these Ready to Go courses include a course-level “Getting Ready” diagnostic that helps pinpoint student weaknesses in prerequisite skills. Ask your Pearson representative about the details for this particular course or to see a copy of this course.

MathXL[®] Online Course (access code required)

MathXL[®] is the homework and assessment engine that runs MyMathLab. (MyMathLab is MathXL plus a learning management system.)

With MathXL, instructors can:

- Create, edit, and assign online homework and tests using algorithmically generated exercises correlated at the objective level to the textbook.
- Create and assign their own online exercises and import TestGen tests for added flexibility.
- Maintain records of all student work tracked in MathXL’s online gradebook.

With MathXL, students can:

- Take chapter tests in MathXL and receive personalized study plans and/or personalized homework assignments based on their test results.
- Use the study plan and/or the homework to link directly to tutorial exercises for the objectives they need to study.
- Access supplemental animations and video clips directly from selected exercises.

MathXL is available to qualified adopters. For more information, visit our website at www.mathxl.com, or contact your Pearson representative.

New! Video Lectures with Optional Subtitles

The video lectures for this text are available in MyMathLab, making it easy and convenient for students to watch the videos from computers at home or on campus, or from their smart phones! The videos feature engaging chapter summaries and worked-out examples. The videos have **optional English subtitles**; they can easily be turned on or off for individual student needs.

PowerPoint[®] Lecture Slides

These slides present key concepts and definitions from the text. They are available in MyMathLab or at www.pearsonhighered.com/irc.

TestGen®

TestGen® (www.pearsoned.com/testgen) enables instructors to build, edit, print, and administer tests using a computerized bank of questions developed to cover all the objectives of the text. TestGen is algorithmically based, allowing instructors to create multiple but equivalent versions of the same question or test with the click of a button. Instructors can also modify test bank questions or add new questions. The software and testbank are available for download from Pearson Education's online catalog.

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*Thomas W. Hungerford
John P. Holcomb, Jr.
Bernadette Mullins*

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To the Student

The key to succeeding in this course is to remember that *mathematics is not a spectator sport*. You can't expect to learn mathematics without *doing* mathematics any more than you could learn to swim without getting wet. You must take an active role, making use of all the resources at your disposal: your instructor, your fellow students, this book, and the supplements that accompany it. Following are some general tips on how to be successful in the course, and some specific tips on how to get the most out of this text and supplementary resources.

Ask Questions! Remember the words of the great Hillel: “The bashful do not learn.” There is no such thing as a “dumb question” (assuming, of course, that you have read the book and your class notes and attempted the homework). Your instructor will welcome questions that arise from a serious effort on your part. So get your money's worth: Ask questions!

Read the Book Interactively! There is more to a math textbook than just the exercise sets. Each section introduces topics carefully with many examples—both mathematical and contextual. Take note of the “Caution” and “Note” comments, and bookmark pages with key definitions or formulas. After reading the example, try the **Checkpoint Exercise** that follows it in the margin to check your understanding of the concept. This will help you solidify your understanding or diagnose if you do not fully understand the concept. The answers to the Checkpoint exercises are right after the homework exercises in each section. Resist the temptation to flip to the answer until you've worked the problem completely!

Take Advantage of the Supplementary Material! Both within and outside the text, there are many resources at your disposal. Take the time to interact with them and determine which resources suit your learning style the best.

- If your instructor allows the use of graphing calculators and/or spreadsheets, work through the examples and exercises marked with the . Some instructors may make this material part of the course, whereas others will not require you to use technology. For those who use technology, there is a *Graphing Calculator Appendix* that covers the basics of calculator use and provides a number of helpful programs for working with some of the topics in the text. An outline of the appendix is on page 859, and the full appendix is available in MyMathLab. In addition, there are **Technology Tips** throughout the text that describe the proper menu or keys to use for various procedures on a graphing calculator. Note that Technology Tips for the TI-84+ C also apply to TI-83+, TI-Nspire, and usually TI-83.
- MyMathLab has a variety of types of resources at your disposal including videos and PowerPoints for every section of the text, example-level videos, animations to help visualize difficult concepts, unlimited practice and assessment on newly learned or prerequisite skills, and access to the *Student Solutions Manual*, *Graphing Calculator Manual* and *Excel Spreadsheet Manual*, a variety of helpful reference cards, and links to useful websites like Texas Instruments.

Do Your Homework! Whether it is paper/pencil homework or assigned online, you must practice what you have learned. Remember, math is not a spectator sport! This is your opportunity to practice those essential skills needed for passing this course, and for applying in future courses or your career.

We wish you the best in your efforts with this course, in future courses, and beyond school.

Prerequisite Skills Test*

The following test is unlike your typical math test. Rather than testing your skills after you have worked on them, this test assesses skills that you should know from previous coursework and will use in this class. It is intended to diagnose any areas that you may need to remediate. Take advantage of the results of this test by checking your answers in Appendix D. The full solutions are included to remind you of the steps to answer the problem.

Find the most simplified answer for the given problems involving fractions:

1. $\frac{5}{2} - 6 =$

2. $\frac{1}{2} \div \frac{2}{5} =$

3. $\frac{1}{3} \div 3 =$

Simplify the given expression, keeping in mind the appropriate order of operations:

4. $7 + 2 - 3(2 \div 6) =$

5. $\frac{2 \times 3 + 12}{1 + 5} - 1 =$

Indicate whether each of the statements is true or false:

6. $\frac{4 + 3}{3} = 5$

7. $\frac{5}{7} + \frac{7}{5} = 1$

8. $\frac{3}{5} + 1 = \frac{6}{5}$

Translate each of the following written expressions into a corresponding mathematical statement. If possible, solve for the unknown value or values.

9. Alicia has n pairs of shoes. Manuel has two more pairs of shoes than Alicia. How many pairs of shoes does Manuel have?
10. David's age and Selina's age, when added together, equals 42. Selena is 6 years older than David. What are David's and Selina's ages?

Solve the following problem.

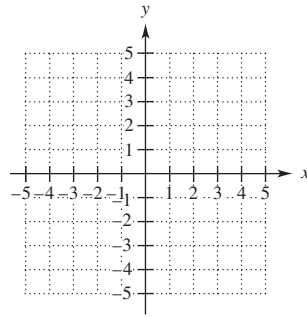
11. The price of a sweater, originally sold for \$72, is reduced by 20%. What is the new sale price of the sweater?

*Full Solutions to this test are provided in the back of the text.

Given the following rectangular coordinate system, graph and label the following points:

12. A: $(3, -2)$ B: $(4, 0)$ C: $(-2, -3)$

13. D: $(3, 5)$ E: $(-1, 4)$ F: $(-4, -5)$



Round the following values as indicated:

14. (a) 4.27659 to the nearest tenth

(b) 245.984 to the nearest unit (whole number)

15. (a) 16.38572 to the nearest hundredth

(b) 1,763,304.42 to the nearest thousand

Write the number that corresponds with the given numerical statement:

16. (a) The Company's liabilities totaled 34 million dollars.

(b) The total of investments was 2.2 thousand dollars.

17. (a) The population of a country is 17 hundred thousand.

(b) The cost of the new airport could run as high as three and a quarter billion dollars.

Answer the following. If there is no solution, indicate that there is no solution and provide a reason.

18. $\frac{5}{0} =$

19. A car is traveling 60 miles per hour. Is the car traveling at 1 mile per minute?

20. Which number is greater, -9 or -900 ?

Calculus Readiness Test*

1. Subtract the polynomials: $(7x^4 - 8x^3 + 2x^2 - 3) - (9x^4 - 2x^2 + 5x + 6)$.
2. Add the rational functions: $\frac{-6}{3x + 4} + \frac{2}{x}$.
3. Find the product: $(3x + 7)(5x - 8)$.
4. Find the product: $(7x + 3)(6x^2 + x - 8)$.
5. Simplify: $\frac{(x^5y^{-5})^3}{(x^2\sqrt{y})^6}$.
6. Multiply out and simplify: $(8x - 9)^2$.
7. Factor: $9x^2 - 49$.
8. Solve for x : $9x - 4 = 8 + 7(x - 5)$.
9. Solve for x : $3x^2 + 2x + 8 = 20x - 7$.
10. Solve the inequality for x : $x^2 - x - 6 < 0$.
11. Solve for x : $5e^{x-3} - 1 = 9$.
12. Solve for x : $\ln(5x + 1) = 2$.
13. Given that $f(x) = 5x^3 - 3x^2 + 9x + 60$, find $f(-2)$.
14. Given that $f(x) = 6x^2 - 7x - 20$, solve $f(x) = 0$.
15. Find an equation of a line through the point $(8, -3)$ with slope -5 .

*Full Solutions to this test are provided in the back of the text.



Algebra and Equations

1

CHAPTER

CHAPTER OUTLINE

- 1.1 The Real Numbers
- 1.2 Polynomials
- 1.3 Factoring
- 1.4 Rational Expressions
- 1.5 Exponents and Radicals
- 1.6 First-Degree Equations
- 1.7 Quadratic Equations

CASE STUDY 1

Consumers Often Need to Just Do the Math

Mathematics is widely used in business, finance, and the biological, social, and physical sciences, from developing efficient production schedules for a factory to mapping the human genome. Mathematics also plays a role in determining interest on a loan from a bank, the growth of traffic on websites, and the study of falling objects. See Exercises 61 and 67 on page 51 and Exercise 63 on page 59.

Algebra and equations are the basic mathematical tools for handling many applications. Your success in this course will depend on your having the algebraic skills presented in this chapter.

1.1 The Real Numbers

Only real numbers will be used in this book.* The names of the most common types of real numbers are as follows.

The Real Numbers

Natural (counting) numbers 1, 2, 3, 4, . . .

Whole numbers 0, 1, 2, 3, 4, . . .

*Not all numbers are real numbers. For example, $\sqrt{-1}$ is a number that is *not* a real number.

Integers	$\dots, -3, -2, -1, 0, 1, 2, 3, \dots$
Rational numbers	All numbers that can be written in the form p/q , where p and q are integers and $q \neq 0$
Irrational numbers	Real numbers that are not rational

As you can see, every natural number is a whole number, and every whole number is an integer. Furthermore, every integer is a rational number. For instance, the integer 7 can be written as the fraction $\frac{7}{1}$ and is therefore a rational number.

One example of an irrational number is π , the ratio of the circumference of a circle to its diameter. The number π can be approximated as $\pi \approx 3.14159$ (\approx means “is approximately equal to”), but there is no rational number that is exactly equal to π .

Example 1

What kind of number is each of the following?

(a) 6

Solution The number 6 is a natural number, a whole number, an integer, a rational number, and a real number.

(b) $\frac{3}{4}$

Solution This number is rational and real.

(c) 3π

Solution Because π is not a rational number, 3π is irrational and real. 

✓ Checkpoint 1

Name all the types of numbers that apply to the following.

(a) -2

(b) $-5/8$

(c) $\pi/5$

Answers to Checkpoint exercises are found at the end of the section.

All real numbers can be written in decimal form. A rational number, when written in decimal form, is either a terminating decimal, such as .5 or .128, or a repeating decimal, in which some block of digits eventually repeats forever, such as 1.3333... or 4.7234234234...[†] Irrational numbers are decimals that neither terminate nor repeat.

When a calculator is used for computations, the answers it produces are often decimal *approximations* of the actual answers; they are accurate enough for most applications. To ensure that your final answer is as accurate as possible,

you should not round off any numbers during long calculator computations.

It is usually OK to round off the final answer to a reasonable number of decimal places once the computation is finished.

The important basic properties of the real numbers are as follows.

Properties of the Real Numbers

For all real numbers, a , b , and c , the following properties hold true:

Commutative properties $a + b = b + a$ and $ab = ba$.

Associative properties $(a + b) + c = a + (b + c)$ and $(ab)c = a(bc)$.

^{*}The use of Checkpoint exercises is explained in the “To the Student” section preceding this chapter.

[†]Some graphing calculators have a FRAC key that automatically converts some repeating decimals to fraction form.

Identity properties

There exists a unique real number 0, called the **additive identity**, such that

$$a + 0 = a \quad \text{and} \quad 0 + a = a.$$

There exists a unique real number 1, called the **multiplicative identity**, such that

$$a \cdot 1 = a \quad \text{and} \quad 1 \cdot a = a.$$

Inverse properties

For each real number a , there exists a unique real number $-a$, called the **additive inverse** of a , such that

$$a + (-a) = 0 \quad \text{and} \quad (-a) + a = 0.$$

If $a \neq 0$, there exists a unique real number $1/a$, called the **multiplicative inverse** of a , such that

$$a \cdot \frac{1}{a} = 1 \quad \text{and} \quad \frac{1}{a} \cdot a = 1.$$

Distributive property

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca.$$

The next five examples illustrate the properties listed in the preceding box.

Example 2

The commutative property says that the order in which you add or multiply two quantities doesn't matter.

$$(a) \quad (6 + x) + 9 = 9 + (6 + x) = 9 + (x + 6) \qquad (b) \quad 5 \cdot (9 \cdot 8) = (9 \cdot 8) \cdot 5$$

 **Checkpoint 2**

Name the property illustrated in each of the following examples.

- (a) $(2 + 3) + 9 = (3 + 2) + 9$
- (b) $(2 + 3) + 9 = 2 + (3 + 9)$
- (c) $(2 + 3) + 9 = 9 + (2 + 3)$
- (d) $(4 \cdot 6)p = (6 \cdot 4)p$
- (e) $4(6p) = (4 \cdot 6)p$

 **Checkpoint 3**

Name the property illustrated in each of the following examples.

- (a) $2 + 0 = 2$
- (b) $-\frac{1}{4} \cdot (-4) = 1$
- (c) $-\frac{1}{4} + \frac{1}{4} = 0$
- (d) $1 \cdot \frac{2}{3} = \frac{2}{3}$

Example 3

When the associative property is used, the order of the numbers does not change, but the placement of parentheses does.

$$(a) \quad 4 + (9 + 8) = (4 + 9) + 8 \qquad (b) \quad 3(9x) = (3 \cdot 9)x \quad \checkmark_2$$

Example 4

By the identity properties,

$$(a) \quad -8 + 0 = -8 \qquad (b) \quad (-9) \cdot 1 = -9.$$



TECHNOLOGY TIP To enter -8 on a calculator, use the negation key (labeled $(-)$ or $+/-$), not the subtraction key. On most one-line scientific calculators, key in $8 +/-$. On graphing calculators or two-line scientific calculators, key in either $(-)$ 8 or $+/-$ 8.


Example 5

By the inverse properties, the statements in parts (a) through (d) are true.

$$(a) \quad 9 + (-9) = 0 \qquad (b) \quad -15 + 15 = 0$$

$$(c) \quad -8 \cdot \left(\frac{1}{-8}\right) = 1 \qquad (d) \quad \frac{1}{\sqrt{5}} \cdot \sqrt{5} = 1$$



NOTE There is no real number x such that $0 \cdot x = 1$, so 0 has no multiplicative inverse. 

✓ Checkpoint 4

Use the distributive property to complete each of the following.

- (a) $4(-2 + 5)$
- (b) $2(a + b)$
- (c) $-3(p + 1)$
- (d) $(8 - k)m$
- (e) $5x + 3x$

Example 6 By the distributive property,

- (a) $9(6 + 4) = 9 \cdot 6 + 9 \cdot 4$
- (b) $3(x + y) = 3x + 3y$
- (c) $-8(m + 2) = (-8)(m) + (-8)(2) = -8m - 16$
- (d) $(5 + x)y = 5y + xy$. ✓₄

Order of Operations

Some complicated expressions may contain many sets of parentheses. To avoid ambiguity, the following procedure should be used.

Parentheses

Work separately above and below any fraction bar. Within each set of parentheses or square brackets, start with the innermost set and work outward.

Example 7 Simplify: $[(3 + 2) - 7]5 + 2[(6 \cdot 3) - 13]$.

Solution On each segment, work from the inside out:

$$\begin{aligned} & [(3 + 2) - 7]5 - 2[(6 \cdot 3) - 13] \\ &= [5 - 7]5 + 2(18 - 13) \\ &= [-2]5 + 2(5) \\ &= -10 + 10 = 0. \end{aligned}$$

Does the expression $2 + 4 \times 3$ mean

$$(2 + 4) \times 3 = 6 \times 3 = 18?$$

Or does it mean

$$2 + (4 \times 3) = 2 + 12 = 14?$$

To avoid this ambiguity, mathematicians have adopted the following rules (which are also followed by almost all scientific and graphing calculators).

Order of Operations

1. Find all powers and roots, working from left to right.
2. Do any multiplications or divisions in the order in which they occur, working from left to right.
3. Finally, do any additions or subtractions in the order in which they occur, working from left to right.

If sets of parentheses or square brackets are present, use the rules in the preceding box within each set, working from the innermost set outward.


According to these rules, multiplication is done *before* addition, so $2 + 4 \times 3 = 2 + 12 = 14$. Here are some additional examples.

Example 8 Use the order of operations to evaluate each expression if $x = -2$, $y = 5$, and $z = -3$.

(a) $-4x^2 - 7y + 4z$

Solution Use parentheses when replacing letters with numbers:

$$\begin{aligned} -4x^2 - 7y + 4z &= -4(-2)^2 - 7(5) + 4(-3) \\ &= -4(4) - 7(5) + 4(-3) = -16 - 35 - 12 = -63. \end{aligned}$$

(b) $\frac{2(x - 5)^2 + 4y}{z + 4} = \frac{2(-2 - 5)^2 + 4(5)}{-3 + 4}$
 $= \frac{2(-7)^2 + 20}{1}$
 $= 2(49) + 20 = 118.$ 

 **Checkpoint 5**

Evaluate the following if $m = -5$ and $n = 8$.

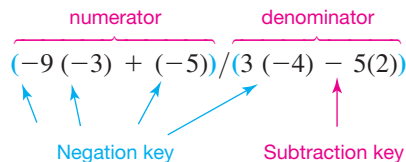
(a) $-2mn - 2m^2$


(b) $\frac{4(n - 5)^2 - m}{m + n}$

Example 9 Use a calculator to evaluate

$$\frac{-9(-3) + (-5)}{3(-4) - 5(2)}$$

Solution Use extra parentheses (shown here in blue) around the numerator and denominator when you enter the number in your calculator, and be careful to distinguish the negation key from the subtraction key.



If you don't get -1 as the answer, then you are entering something incorrectly. 

 **Checkpoint 6**

Use a calculator to evaluate the following.

(a) $4^2 \div 8 + 3^2 \div 3$

(b) $[-7 + (-9)] \cdot (-4) - 8(3)$

(c) $\frac{-11 - (-12) - 4 \cdot 5}{4(-2) - (-6)(-5)}$

(d) $\frac{36 \div 4 \cdot 3 \div 9 + 1}{9 \div (-6) \cdot 8 - 4}$

 **TECHNOLOGY TIP**

On one-line scientific calculators, $\sqrt{40}$ is entered as $40 \sqrt{}$. On graphing calculators and two-line scientific calculators, key in $\sqrt{40}$ ENTER (or EXE).

Square Roots

There are two numbers whose square is 16, namely, 4 and -4 . The positive one, 4, is called the **square root** of 16. Similarly, the square root of a nonnegative number d is defined to be the **nonnegative** number whose square is d ; this number is denoted \sqrt{d} . For instance,

$$\begin{aligned} \sqrt{36} &= 6 \text{ because } 6^2 = 36, & \sqrt{0} &= 0 \text{ because } 0^2 = 0, \text{ and} \\ \sqrt{1.44} &= 1.2 \text{ because } (1.2)^2 = 1.44. \end{aligned}$$

No negative number has a square root that is a real number. For instance, there is no real number whose square is -4 , so -4 has no square root.

Every nonnegative real number has a square root. Unless an integer is a perfect square (such as $64 = 8^2$), its square root is an irrational number. A calculator can be used to obtain a rational approximation of these square roots.

Example 10 Estimate each of the given quantities. Verify your estimates with a calculator.

(a) $\sqrt{40}$

Solution Since $6^2 = 36$ and $7^2 = 49$, $\sqrt{40}$ must be a number between 6 and 7. A typical calculator shows that $\sqrt{40} \approx 6.32455532$.

✓ Checkpoint 7

Estimate each of the following.

- (a) $\sqrt{73}$
 (b) $\sqrt{22} + 3$
 (c) Confirm your estimates in parts (a) and (b) with a calculator.

✓ Checkpoint 8

Draw a number line, and graph the numbers -4 , -1 , 0 , 1 , 2.5 , and $13/4$ on it.

(b) $5\sqrt{7}$

Solution $\sqrt{7}$ is between 2 and 3 because $2^2 = 4$ and $3^2 = 9$, so $5\sqrt{7}$ must be a number between $5 \cdot 2 = 10$ and $5 \cdot 3 = 15$. A calculator shows that $5\sqrt{7} \approx 13.22875656$. ✓₇

CAUTION If c and d are positive real numbers, then $\sqrt{c+d}$ is *not* equal to $\sqrt{c} + \sqrt{d}$. For example, $\sqrt{9+16} = \sqrt{25} = 5$, but $\sqrt{9} + \sqrt{16} = 3 + 4 = 7$.

The Number Line

The real numbers can be illustrated geometrically with a diagram called a **number line**. Each real number corresponds to exactly one point on the line and vice versa. A number line with several sample numbers located (or **graphed**) on it is shown in Figure 1.1. ✓₈

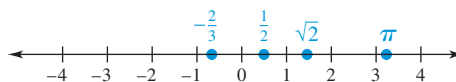


Figure 1.1

When comparing the sizes of two real numbers, the following symbols are used.

Symbol	Read	Meaning
$a < b$	a is less than b .	a lies to the <i>left</i> of b on the number line.
$b > a$	b is greater than a .	b lies to the <i>right</i> of a on the number line.

Note that $a < b$ means the same thing as $b > a$. The inequality symbols are sometimes joined with the equals sign, as follows.

Symbol	Read	Meaning
$a \leq b$	a is less than or equal to b .	either $a < b$ or $a = b$
$b \geq a$	b is greater than or equal to a .	either $b > a$ or $b = a$

TECHNOLOGY TIP

If your graphing calculator has inequality symbols (usually located on the TEST menu), you can key in statements such as “ $5 < 12$ ” or “ $-2 \geq 3$.” When you press ENTER, the calculator will display 1 if the statement is true and 0 if it is false.

Only one part of an “either . . . or” statement needs to be true for the entire statement to be considered true. So the statement $3 \leq 7$ is true because $3 < 7$, and the statement $3 \leq 3$ is true because $3 = 3$.

Example 11 Write *true* or *false* for each of the following.

(a) $8 < 12$

Solution This statement says that 8 is less than 12, which is true.

(b) $-6 > -3$

Solution The graph in Figure 1.2 shows that -6 is to the *left* of -3 . Thus, $-6 < -3$, and the given statement is false.



Figure 1.2

(c) $-2 \leq -2$

Solution Because $-2 = -2$, this statement is true. ✓₉

✓ Checkpoint 9

Write *true* or *false* for the following.

- (a) $-9 \leq -2$
 (b) $8 > -3$
 (c) $-14 \leq -20$

A number line can be used to draw the graph of a set of numbers, as shown in the next few examples.

Example 12 Graph all real numbers x such that $1 < x < 5$.

Solution This graph includes all the real numbers between 1 and 5, not just the integers. Graph these numbers by drawing a heavy line from 1 to 5 on the number line, as in Figure 1.3. Parentheses at 1 and 5 show that neither of these points belongs to the graph.

Checkpoint 10

Graph all real numbers x such that

- (a) $-5 < x < 1$
- (b) $4 < x < 7$.



Figure 1.3

A set that consists of all the real numbers between two points, such as $1 < x < 5$ in Example 12, is called an **interval**. A special notation called **interval notation** is used to indicate an interval on the number line. For example, the interval including all numbers x such that $-2 < x < 3$ is written as $(-2, 3)$. The parentheses indicate that the numbers -2 and 3 are *not* included. If -2 and 3 are to be included in the interval, square brackets are used, as in $[-2, 3]$. The following chart shows several typical intervals, where $a < b$.

Intervals

Inequality	Interval Notation	Explanation
$a \leq x \leq b$	$[a, b]$	Both a and b are included.
$a \leq x < b$	$[a, b)$	a is included; b is not.
$a < x \leq b$	$(a, b]$	b is included; a is not.
$a < x < b$	(a, b)	Neither a nor b is included.

Interval notation is also used to describe sets such as the set of all numbers x such that $x \geq -2$. This interval is written $[-2, \infty)$. The set of all real numbers is written $(-\infty, \infty)$ in interval notation.

Example 13 Graph the interval $[-2, \infty)$.

Solution Start at -2 and draw a heavy line to the right, as in Figure 1.4. Use a square bracket at -2 to show that -2 itself is part of the graph. The symbol ∞ , read “infinity,” *does not* represent a number. This notation simply indicates that *all* numbers greater than -2 are in the interval. Similarly, the notation $(-\infty, 2)$ indicates the set of all numbers x such that $x < 2$.

Checkpoint 11

Graph all real numbers x in the given interval.

- (a) $(-\infty, 4]$
- (b) $[-2, 1]$

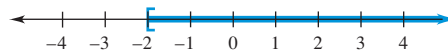


Figure 1.4

Absolute Value

The **absolute value** of a real number a is the distance from a to 0 on the number line and is written $|a|$. For example, Figure 1.5 shows that the distance from 9 to 0 on the number line is 9, so we have $|9| = 9$. The figure also shows that $|-9| = 9$, because the distance from -9 to 0 is also 9.

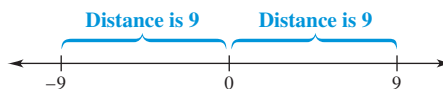


Figure 1.5

The facts that $|9| = 9$ and $|-9| = 9 = -(-9)$ suggest the following algebraic definition of absolute value.

Absolute Value

For any real number a ,

$$\begin{aligned} |a| &= a && \text{if } a \geq 0 \\ |a| &= -a && \text{if } a < 0. \end{aligned}$$

The first part of the definition shows that $|0| = 0$ (because $0 \geq 0$). It also shows that the absolute value of any positive number a is the number itself, so $|a|$ is positive in such cases. The second part of the definition says that the absolute value of a negative number a is the *negative* of a . For instance, if $a = -5$, then $|-5| = -(-5) = 5$. So $|-5|$ is positive. The same thing works for any negative number—that is, its absolute value (the negative of a negative number) is positive. Thus, we can state the following:

For every nonzero real number a , the number $|a|$ is positive.

✓ Checkpoint 12

Find the following.

- $|-6|$
- $-|7|$
- $-|-2|$
- $|-3 - 4|$
- $|2 - 7|$

Example 14 Evaluate $|8 - 9|$.

Solution First, simplify the expression within the absolute-value bars:

$$|8 - 9| = |-1| = 1. \quad \checkmark_{12}$$

1.1 Exercises

In Exercises 1 and 2, label the statement true or false. (See Example 1.)

- Every integer is a rational number.
- Every real number is an irrational number.
- The decimal expansion of the irrational number π begins 3.141592653589793 Use your calculator to determine which of the following rational numbers is the best approximation for the irrational number π :

$$\frac{22}{7}, \quad \frac{355}{113}, \quad \frac{103,993}{33,102}, \quad \frac{2,508,429,787}{798,458,000}$$

Your calculator may tell you that some of these numbers are equal to π , but that just indicates that the number agrees with π for as many decimal places as your calculator can handle (usually 10–14). No rational number is exactly equal to π .

Identify the properties that are illustrated in each of the following. (See Examples 2–6.)

- $0 + (-7) = -7 + 0$
- $6(t + 4) = 6t + 6 \cdot 4$

6. $3 + (-3) = (-3) + 3$

7. $-5 + 0 = -5$

8. $(-4) \cdot \left(\frac{-1}{4}\right) = 1$

9. $8 + (12 + 6) = (8 + 12) + 6$

10. $1 \cdot (-20) = -20$

11. How is the additive inverse property related to the additive identity property? the multiplicative inverse property to the multiplicative identity property?

12. Explain the distinction between the commutative and associative properties.

Evaluate each of the following if $p = -2$, $q = 3$, and $r = -5$. (See Examples 7–9.)

13. $-3(p + 5q)$

14. $2(q - r)$

15. $\frac{q + r}{q + p}$

16. $\frac{3q}{3p - 2r}$

Business The nominal annual percentage rate (APR) reported by lenders has the formula $APR = 12r$, where r is the monthly interest rate. Find the APR when

17. $r = 3.8$ 18. $r = 0.8$

Find the monthly interest rate r when

19. $APR = 11$ 20. $APR = 13.2$

Evaluate each expression, using the order of operations given in the text. (See Examples 7–9.)

21. $3 - 4 \cdot 5 + 5$ 22. $8 - (-4)^2 - (-12)$
 23. $(4 - 5) \cdot 6 + 6$ 24. $\frac{2(3 - 7) + 4(8)}{4(-3) + (-3)(-2)}$
 25. $8 - 4^2 - (-12)$
 26. $-(3 - 5) - [2 - (3^2 - 13)]$
 27. $\frac{2(-3) + 3/(-2) - 2/(-\sqrt{16})}{\sqrt{64} - 1}$
 28. $\frac{6^2 - 3\sqrt{25}}{\sqrt{6^2 + 13}}$

Use a calculator to help you list the given numbers in order from smallest to largest. (See Example 10.)

29. $\frac{189}{37}$, $\frac{4587}{691}$, $\sqrt{47}$, 6.735, $\sqrt{27}$, $\frac{2040}{523}$
 30. $\frac{385}{117}$, $\sqrt{10}$, $\frac{187}{63}$, π , $\sqrt{\sqrt{85}}$, 2.9884

Express each of the following statements in symbols, using $<$, $>$, \leq , or \geq .

31. 12 is less than 18.5.
 32. -2 is greater than -20 .
 33. x is greater than or equal to 5.7.
 34. y is less than or equal to -5 .
 35. z is at most 7.5.
 36. w is negative.

Fill in the blank with $<$, $=$, or $>$ so that the resulting statement is true.

37. -6 _____ -2 38. $3/4$ _____ $.75$
 39. 3.14 _____ π 40. $1/3$ _____ $.33$

Fill in the blank so as to produce two equivalent statements. For example, the arithmetic statement “ a is negative” is equivalent to the geometric statement “the point a lies to the left of the point 0.”

Arithmetic Statement	Geometric Statement
41. $a \geq b$	_____
42. _____	a lies c units to the right of b
43. _____	a lies between b and c , and to the right of c
44. a is positive	_____

Graph the given intervals on a number line. (See Examples 12 and 13.)

45. $(-8, -1)$ 46. $[-1, 10]$
 47. $(-2, 3]$ 48. $[-2, 2)$
 49. $(-2, \infty)$ 50. $(-\infty, -2]$

Evaluate each of the following expressions (see Example 14).

51. $|-9| - |-12|$
 52. $|8| - |-4|$
 53. $-|-4| - |-1 - 14|$
 54. $-|6| - |-12 - 4|$

In each of the following problems, fill in the blank with either $=$, $<$, or $>$, so that the resulting statement is true.

55. $|5|$ _____ $|-5|$
 56. $-|-4|$ _____ $|4|$
 57. $|10 - 3|$ _____ $|3 - 10|$
 58. $|6 - (-4)|$ _____ $|-4 - 6|$
 59. $|-2 + 8|$ _____ $|2 - 8|$
 60. $|3| \cdot |-5|$ _____ $|3(-5)|$
 61. $|3 - 5|$ _____ $|3| - |5|$
 62. $|-5 + 1|$ _____ $|-5| + |1|$

Write the expression without using absolute-value notation.

63. $|a - 7|$ if $a < 7$ 64. $|b - c|$ if $b \geq c$
 65. If a and b are any real numbers, is it always true that $|a + b| = |a| + |b|$? Explain your answer.
 66. If a and b are any two real numbers, is it always true that $|a - b| = |b - a|$? Explain your answer.
 67. For which real numbers b does $|2 - b| = |2 + b|$? Explain your answer.

68. **Health** Data from the National Health and Nutrition Examination Study estimates that 95% of adult heights (inches) are in the following ranges for females and males. (Data from: www.cdc.gov/nchs/nhanes.htm.)

Females	63.5 ± 8.4
Males	68.9 ± 9.3

Express the ranges as an absolute-value inequality in which x is the height of the person.

Business The Consumer Price Index (CPI) tracks the cost of a typical sample of a consumer goods. The following table shows the percentage increase in the CPI for each year in a 10-year period.

Year	2003	2004	2005	2006	2007
% Increase in CPI	2.3	2.7	2.5	3.2	4.1
Year	2008	2009	2010	2011	2012
% Increase in CPI	0.1	2.7	1.5	3.0	1.7